A Method for Strengthening Prism and Antiprism Stacks

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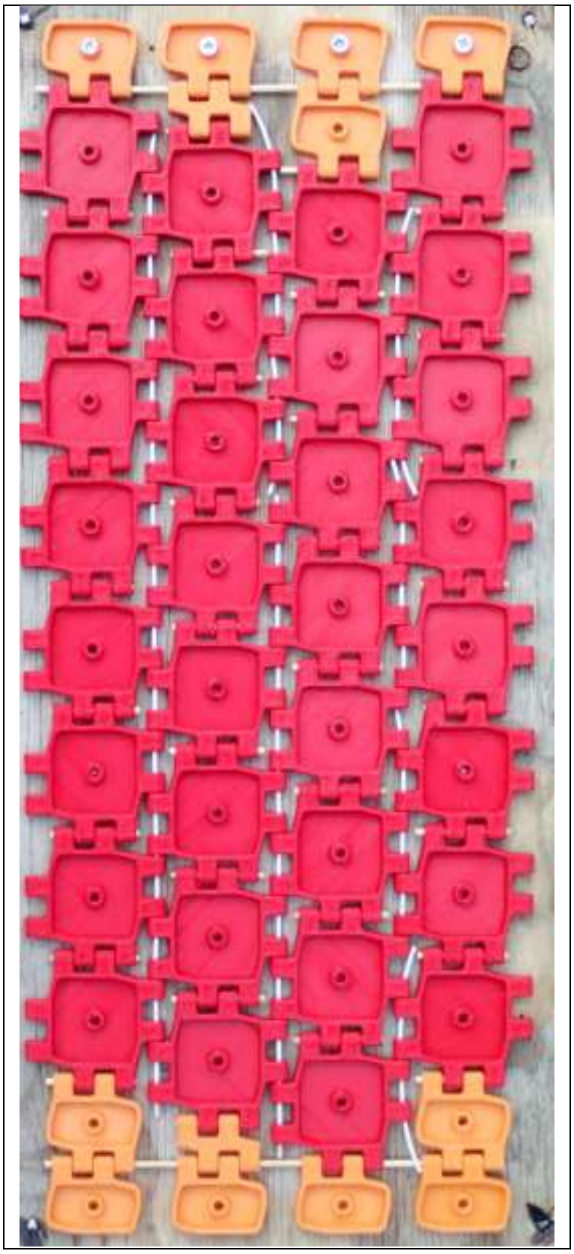


Cover Picture: Flat Version of Model shown in Figure 15.

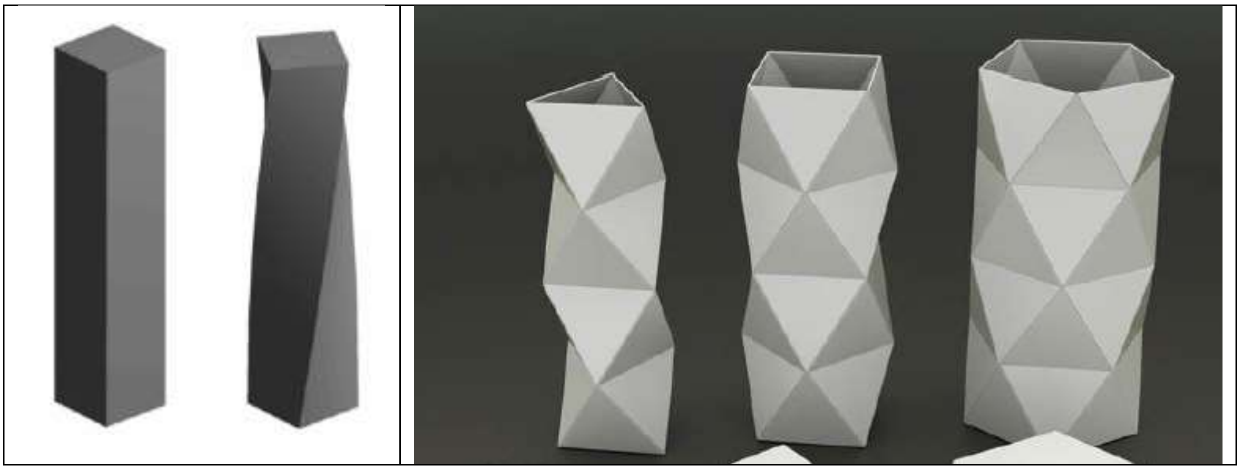


Figure 1, Prismatic Building models from Tang (2013), Antiprism Stacks from Roelofs (2013).

Abstract

Prism and antiprism stacks are cylinder-like polyhedra. When formed from a paper net, the edges between their layers coincide, however when made from a 3d printed construction kit I have developed, stacks can have irregular forms with conflicting layer edges. Due to internal interferences, the forms can have rigidity greater than the regular forms they are based on.

The kit used in this paper has 3d printed plastic face parts joined with timber- and 3d-printer-filament- edges. Faces have two interlocking tabs per edge, and the kit uses optional ways of linking tabs to vary constructions.

The forms depend on basic geometry, can make Euclidian Solids, use 3d printed parts and 3d printer filament and could be used for maths education. The forms and methods of creating strength described here could be used on larger scales to create shelters, sculptures and buildings.

Scope

This article introduces a way of strengthening prism and antiprism stacks through use of a construction kit. While it covers techniques and explanations in a logical progression, it does not set out to be comprehensive.

This article shows two methods of scaling the kit and speculates on large (building size) and small (microscopic) applications for assemblies with induced rigidity. Some model configurations show bistability and polymorphism, but examining these is out of scope. A more general description of the kit and what can be made from it, and all quantitative assessments are out of scope.

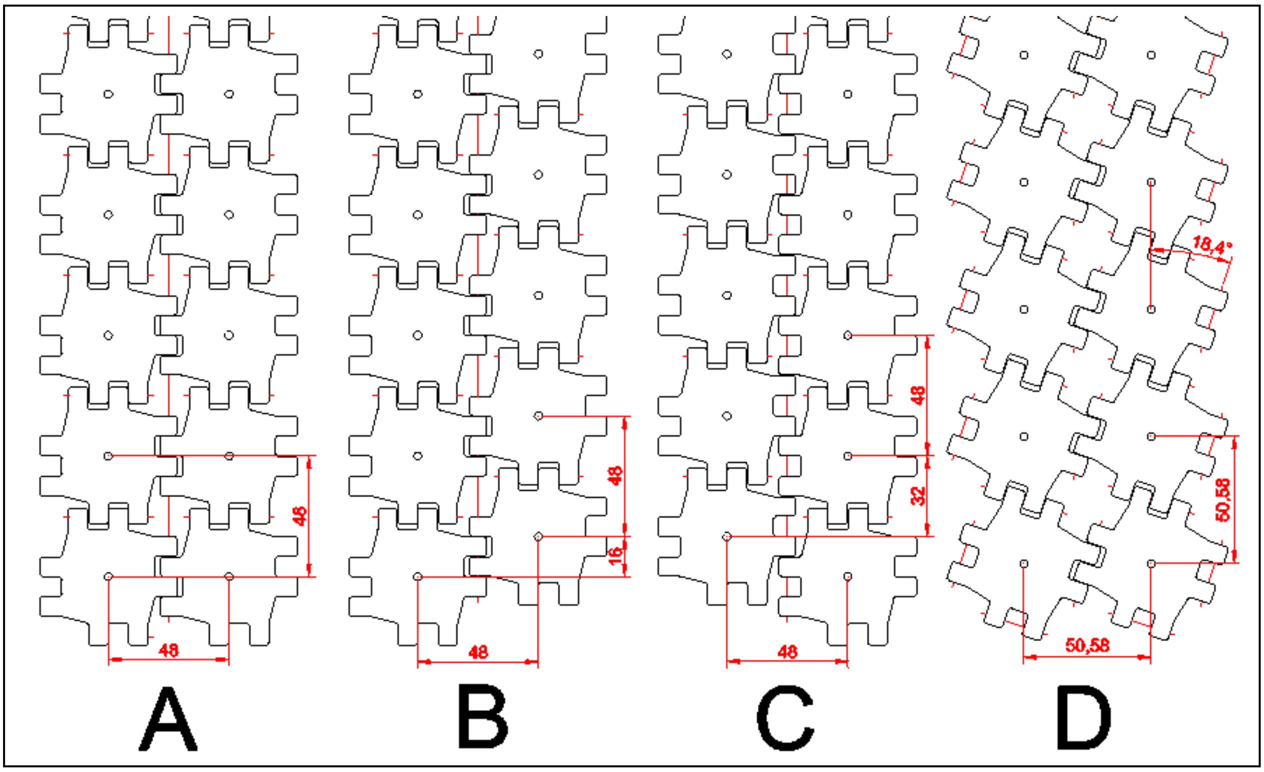
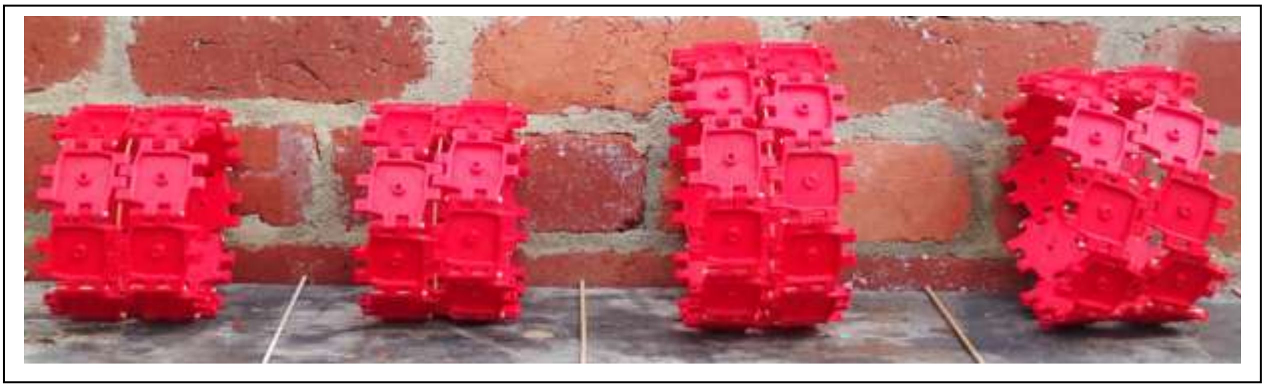


Figure 2, Patterns for linking prism layers, red lines indicate edge materials. a) standard pattern b) misaligned columns with tabs engaged directly c) misaligned columns with tabs engaged in large gap d) Rotated parts: misaligned rows and columns make an expanded pitch pattern.

Prisms

Figure 2 shows nets made from square parts which can form prism stacks by rolling and connecting the narrow ends. The parts allow four ways of securing strips together to form prism stacks, one (fig 2a) producing regular stacks and three (fig 2b, c, d) with adjoining layers misaligned producing irregular stacks. When misaligned tessellations are rolled, they create stronger interference fits between edges and parts, and bend edges and parts, increasing rigidity. This rigidity depends on the parts design, number of sides, join patterns between layers, and the edge material.

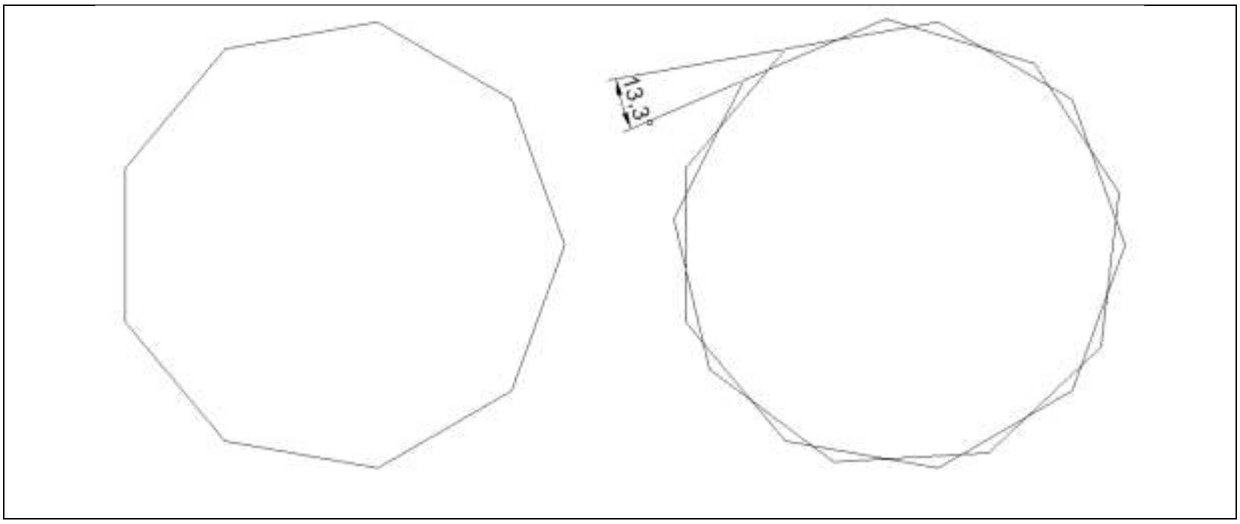


a) *b)* *c)* *d)*

Figure 3, Cylindrical form prism as per fig 2 a, b, c, d with 3mm diameter wooden skewers and 1.7mm 3d printer filament as edges. a) 9 sided b) 9 sided c) 12 sided, d) 10 sided.

a) b) c) d)

Figure 4, Prism stacks from fig. 3 supporting 500g of screws.



a) b)

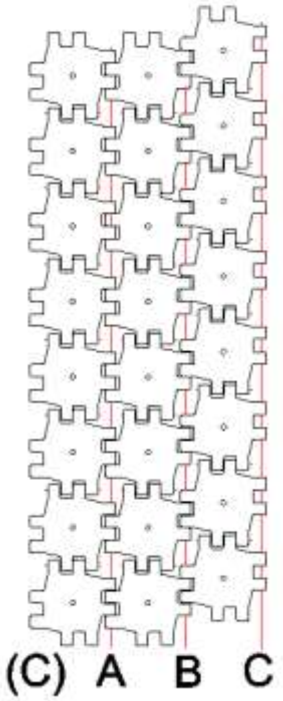
Figure 5, Layer join profiles in a) regular and b) irregular prism stacks.

Figures 3a and 4a show a regular 9-sided prism stack with layer edges coinciding on a polygon as shown in fig 5a. However within irregular stack layer joins (figs 3b, 4b, 3c, 4c), one layer sits with edges on a polygon, and its neighbour sits on a polygon rotated from the first. In the case of figs 3b / 4b, the rotation angle is equivalent to 1/3 of the part pitch, or (360 degrees / (9*3) or) 13.33 degrees (fig 5b). The adjoining layers assume adjacent-layer-influenced compromise positions at the edges.

Figure 6, Prism stacks with 2 layers and fig 2b pattern. Left to right 8, 9, 10 and 11 sided stacks

With these irregular stacks, angles inducing interference fits become larger the fewer sides there are, and stacks become more rigid until assemblies become too tight for assembly (Figs. 6 & 7). Hugh Kenner (2003, 8) provides a formula for angle differences between polygons in geodesic prisms, and this also depends on the number of sides in the polygon.

Figure 7, Loaded with 0.5kg, the fig 6 prism stacks deform more as the number of sides increases.



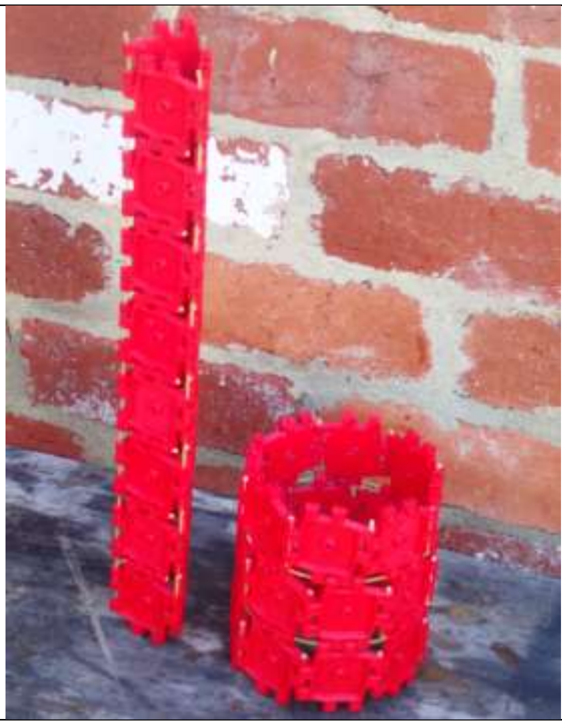
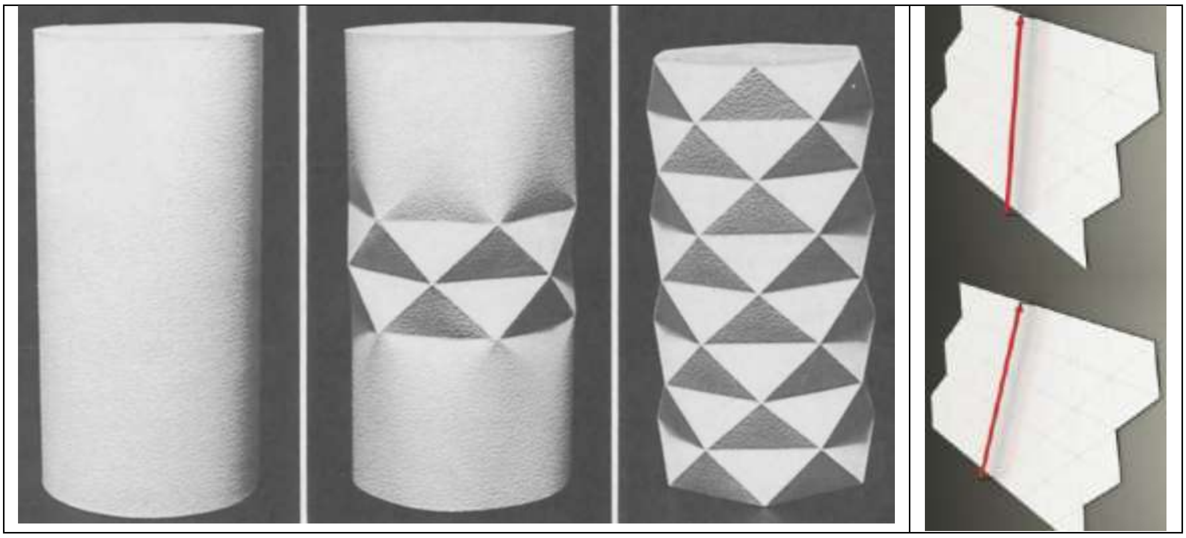


Figure 8, rectangular net plan, and configurations of the net made by joining long / short edges of the net. Join types A, B, C match fig 2.

When prism nets are more than two tiles wide, they can be rolled in two directions as shown in fig 8. In the triangular column the long edge is creased, while in the octagonal / circular column it is rolled. The rigidity of the octagonal column may depend on the joins present inside ie AB as shown, or BC, or CA.

Antiprisms

Antiprisms are made from triangles, and like prisms, they can be made from nets of flat parts. When rolled, the triangular faces settle at distinct angles to each other, a phenomenon observed by Yoshimura (Tarnai 1989) and related to the buckling of cylinders under compression as shown in fig. 9a. Variations in antiprism stacks exist, and Roelofs (2013) uses the phrase “changing the connection” for one of these which creates spirals (fig 9b). As shown in figs. 9 and 11, antiprism stacks can be formed by both non-equilateral and equilateral triangles.



a) ***b)***

Figure 9, a) Yoshimura-pattern cylinder buckling forms antiprism stacks (Tarnai 1989) b) Shifting the point of connection in nets to create spirals (Roelofs 2013)

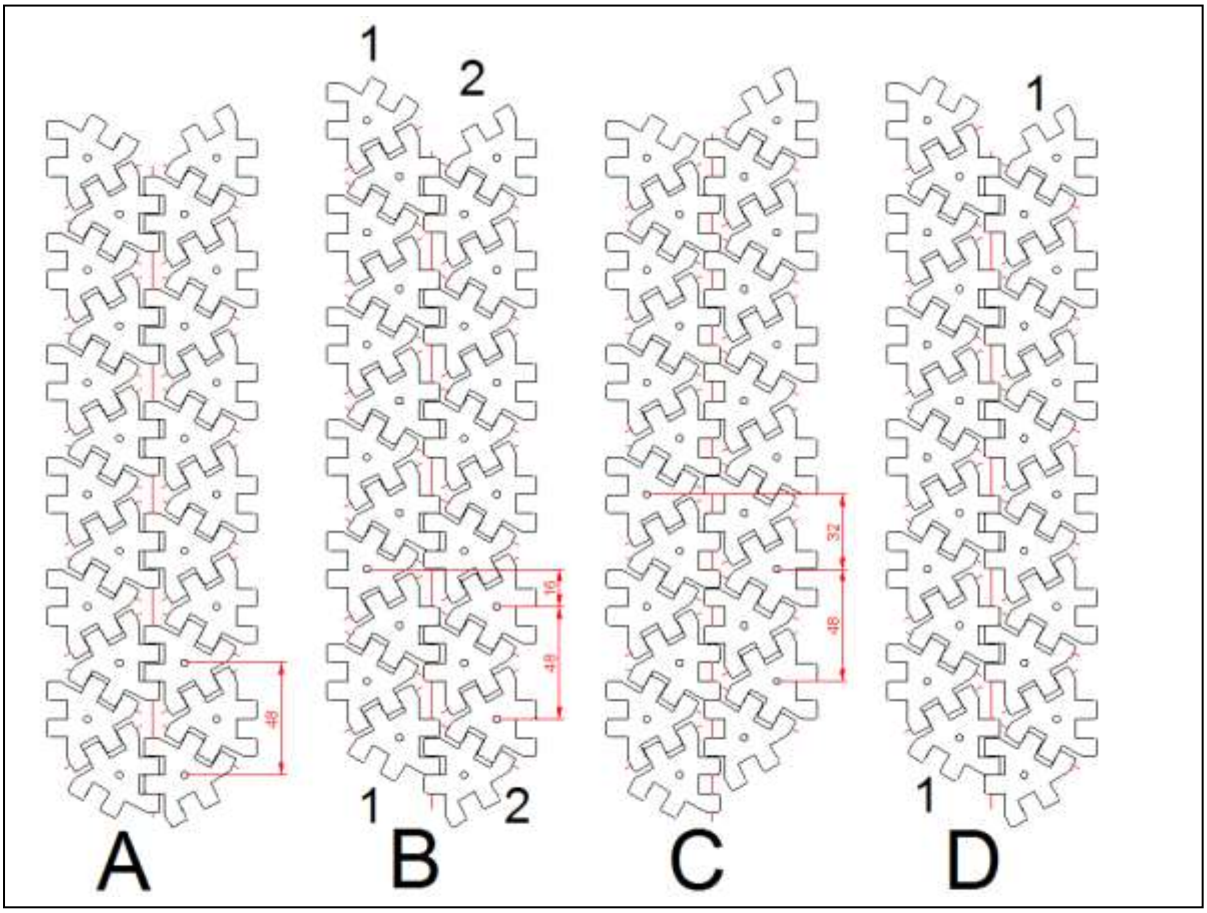


Figure 10, Antiprism nets. a) aligned edges, b) and c), misaligned edges. d) resembles b) but has a changed point of connection (1 connects to 1 on adjacent strips) creating a spiral.

Fig 10 shows antiprism nets: a, b and c have similarities to the prism patterns of fig 2 a, b and c, while fig 10d is a spiral net. Creating mismatches and introducing spirals creates variations influencing rigidity in the rolled forms. Some antiprism stacks are shown in fig. 11.

a) ***b)***

Figure 11, Antiprism and spiral stacks, rolled versions of fig 10 a & d nets

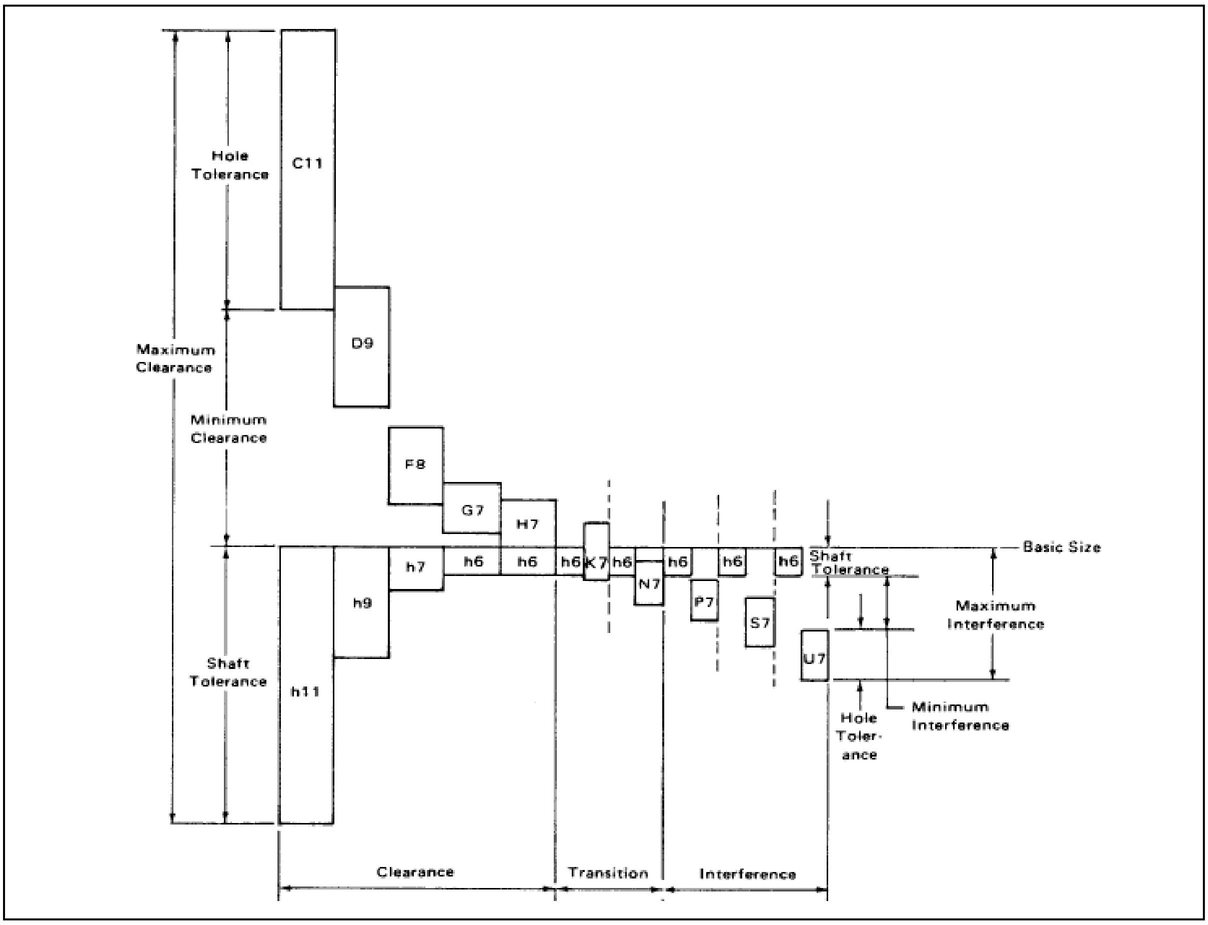


Figure 12, Preferred shaft base fits (ANSI B4.2 from http://www.gometricusa.org/)

Interference Fits

The mechanism for the induced rigidity of stacks such as those in fig 7 is tightening of the gaps between edge materials (1.7mm OD 3d printer filament / 2.8mm OD timber skewers) and parts, (3d printed PLA plastic, 3mm nominal ID holes) and between parts themselves. The clearances between edge materials and holes in parts is large in engineering terms and can be measured in tenths of millimetres whereas standard engineering clearances between fitting metal parts is measured in thousandths of millimetres (gometricusa.org).

Nevertheless, some sense can be made of this interference mechanism by calling it a “shaft basis fit”: the arrangement is similar to sizing a housing to suit a bearing outside diameter where the bearing (edge material) has a known size and the housing (part hole) has a size set by the designer to tolerate a range of operating conditions. As tessellations are rolled to produce stacks, some regions reach fits corresponding to “clearance, transition, interference” classes shown in fig 12.

In engineered components, recognised standards, material grade, geometric and size tolerances, and surface finish complete diameter specifications, and efforts are made to ensure machines used to make parts (ie casting machines, lathes) are capable of producing parts to specification (I.E. Aust. 1982). Because the 3d printing processes used to make parts for this article are less controlled, wide variation in diameter can be expected, meaning loose nominal fits are needed to assemble the parts by hand. The rigidity of a stack results from a range of interference fits at pinch points distributed regularly throughout.

Summary

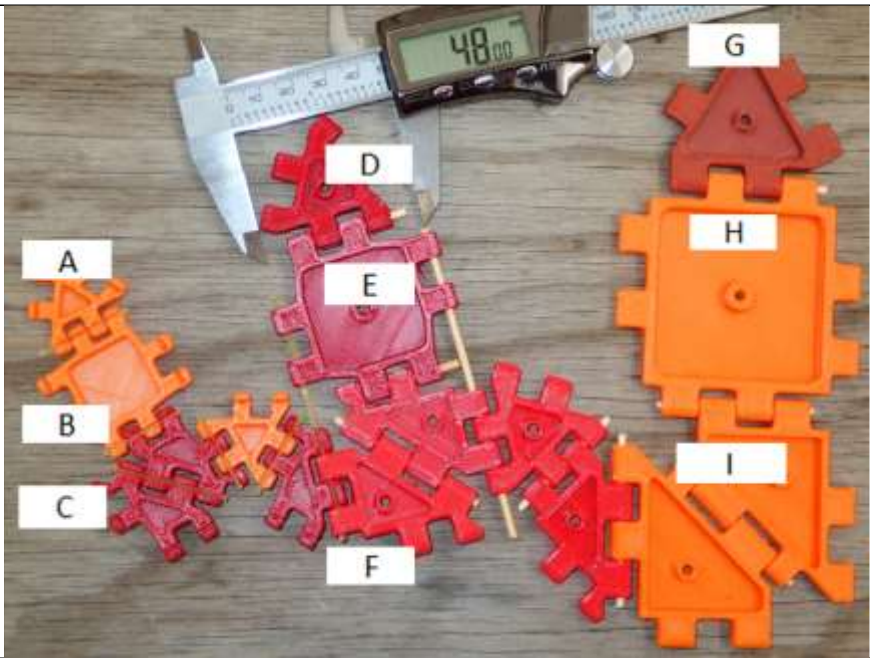
Table 1 shows the various prism and antiprism stacks introduced in this paper. Further variations such as spiral prism structures may be possible. Conflicting edge lines between layers in prism and antiprism stacks create rigidity, and this can be influenced by misalignment, rotation, changing the point of connection of parts, and combinations of these.

	Aligned	Misaligned	Rotated	Spiral
Pattern adds rigidity	No, fig 4	Yes, figs 4, 6	Yes, fig 4	Not Confirmed
Prism	fig 3a	fig 3b,c	fig 3d	
Antiprism	fig 11a	fig 11b		fig 11b

Table 1, Summary

Scalability

At the time of writing, two forms of scalability of the parts series have been examined through printed parts and made objects: a scalable parts series based on kit-compatible right angle isosceles triangles, and furniture based on faces made using plywood and 3d printing.



PITCH#	MINIMUM PITCH= 48mm*(2^0.5)^(PITCH#-2)	EQUILATERAL TRIANGLE	SQUARE	RIGHT ANGLE ISCOSCELES TRIANGLE (MAXIMUM PITCH)	
1	33.9	A	B	C	48.0
2	48.0	D	E	F	67.9
3	67.9	G	H	I	96.0

Table 2, Sampler showing geometric sequence of kit parts

The scalable series is based on right angle triangles with two equal sides and one side 1.41 (or $2^{0.5}$) times longer. These triangles interface with equilateral triangles, and squares and other compatible parts. Table 2 shows parts, summarises their characteristics and shows how parts with different pitches connect through the right angle isosceles triangles.

Figure 13, Table made using 3d printed edge joins.

Larger pieces in the form of timber furniture have been made using geometry principles derived from the kit parts. In these oblong pieces, edges are timber dowel, and faces or their equivalents are from fasteners, routed plywood, and 3d printed parts. An example of a table is shown in fig 13: it doesn't have or need complete sets of faces, edges or edge joins.

Possible Applications

David Eagleman (2017, 155) quotes Thomas Edison's list of possible uses for the phonograph, and Kevin Francis (2013) lists both microscopic and human scale applications for origami folds. Here is a list of possible applications for this rigidity technique:

Building construction, print and preassemble building sections, then assemble pieces on site.

Selectively flexible circular arms for soft robotics.

As modular, interchangeable structural elements or cladding or roofing shade for buildings, possibly including solar panels, glass panels, energy storage panels, open panels, opening panels, water heating panels, acoustic panels or panels for containing soil and irrigation.

Habitable structures for arctic, desert, space or underwater environments where cylinders form closed rings (donut shape)

Semicircular Buildings which can be assembled flat on a slab then erected by pulling the 2 edges (Points of "D") together and become rigid when full height is reached. (Nissen or Quonset Hut Design) In these designs the earth acts as a tension member (Kenner 2003). A demonstration model has been made using rectangular pieces and is shown in fig. 15. Similar designs using triangular pieces would have more diverse face plane directions, with implications for acoustics, or solar panel or window placement.

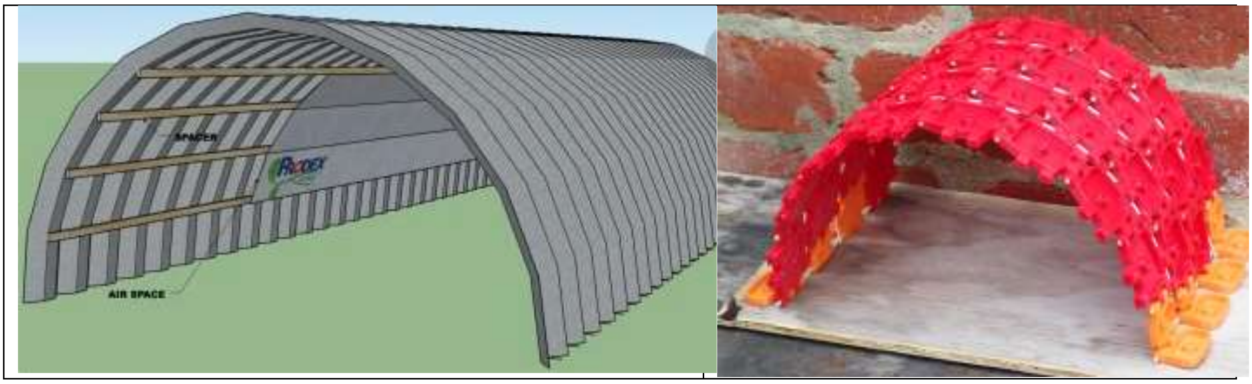


Figure 15, Table made using 3d printed edge joins.

A shell (exoskeleton) for structures where pressure from inside holds an impervious liner against a shell (space station, pressurised aircraft).

A shell (endoskeleton) for structures where pressure from outside holds an impervious liner against a shell (underwater habitations).

Framework for both internal and external skins making an insulated structure.

As a tubular structural power supply with cells in individual panels.

Combine these rigidity methods with tensegrity and geodesic techniques (Kenner, 2003).

Conclusion

The construction kit used here could already allow students to play creatively, promoting engineering, mathematical and manipulative skills. By reproducing the kit samples' rigidity in larger scales, new sculptural forms and effective structural applications could be developed. The patterns shown here could be reproduced or recognised in molecular structures, potentially assisting to make new materials.

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